



# Fox and McDonald's Introduction to Fluid Mechanics

TENTH EDITION

JOHN W. MITCHELL

WILEY



FOX AND MCDONALD'S

# *Introduction to Fluid Mechanics*

10th Edition

**John W. Mitchell** *University of Wisconsin-Madison*

**WILEY**

VICE PRESIDENT AND DIRECTOR Laurie Rosatone  
SENIOR DIRECTOR Don Fowley  
EDITOR Jennifer Brady  
EDITORIAL MANAGER Judy Howarth  
CONTENT MANAGEMENT DIRECTOR Lisa Wojcik  
CONTENT MANAGER Nichole Urban  
SENIOR CONTENT SPECIALIST Nicole Repasky  
PRODUCTION EDITOR Padmapriya Soundararajan  
COVER PHOTO CREDIT © Johner Images/Getty Images, © artjazz/Shutterstock

This book was set in 10/12 STIX SPi Global and printed and bound by Quad Graphics.

Founded in 1807, John Wiley & Sons, Inc. has been a valued source of knowledge and understanding for more than 200 years, helping people around the world meet their needs and fulfill their aspirations. Our company is built on a foundation of principles that include responsibility to the communities we serve and where we live and work. In 2008, we launched a Corporate Citizenship Initiative, a global effort to address the environmental, social, economic, and ethical challenges we face in our business. Among the issues we are addressing are carbon impact, paper specifications and procurement, ethical conduct within our business and among our vendors, and community and charitable support. For more information, please visit our website: [www.wiley.com/go/citizenship](http://www.wiley.com/go/citizenship).

Copyright © 2020, 2015, 2011, 2006 John Wiley & Sons, Inc. All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923 (Web site: [www.copyright.com](http://www.copyright.com)). Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030-5774, (201) 748-6011, fax (201) 748-6008, or online at: [www.wiley.com/go/permissions](http://www.wiley.com/go/permissions).

Evaluation copies are provided to qualified academics and professionals for review purposes only, for use in their courses during the next academic year. These copies are licensed and may not be sold or transferred to a third party. Upon completion of the review period, please return the evaluation copy to Wiley. Return instructions and a free of charge return shipping label are available at: [www.wiley.com/go/returnlabel](http://www.wiley.com/go/returnlabel). If you have chosen to adopt this textbook for use in your course, please accept this book as your complimentary desk copy. Outside of the United States, please contact your local sales representative.

ISBN: 978-1-119-61649-8 (PBK)  
ISBN: 978-1-119-61657-3 (EVALC)

***Library of Congress Cataloging in Publication Data:***

Names: Fox, Robert W., 1934–2014, author. | Mitchell, John W., author.  
Title: Fox and McDonald's introduction to fluid mechanics / John W. Mitchell, University of Wisconsin-Madison.  
Other titles: Introduction to fluid mechanics  
Description: 10th edition. | Hoboken : Wiley, 2020. | Includes bibliographical references and index.  
Identifiers: LCCN 2019041375 (print) | LCCN 2019041376 (ebook) | ISBN 9781119616498 (paperback) | ISBN 9781119616566 (adobe pdf) | ISBN 9781119603764 (epub)  
Subjects: LCSH: Fluid mechanics.  
Classification: LCC TA357 .F69 2020 (print) | LCC TA357 (ebook) | DDC 620.1/06–dc23  
LC record available at <https://lcn.loc.gov/2019041375>  
LC ebook record available at <https://lcn.loc.gov/2019041376>

The inside back cover will contain printing identification and country of origin if omitted from this page. In addition, if the ISBN on the back cover differs from the ISBN on this page, the one on the back cover is correct.

# Preface

## ***Introduction***

This text is written for an introductory course in fluid mechanics. Our approach to the subject emphasizes the physical concepts of fluid mechanics and methods of analysis that begin from basic principles. One primary objective of this text is to help users develop an orderly approach to problem solving. Thus, we always start from governing equations, state assumptions clearly, and try to relate mathematical results to corresponding physical behavior. We emphasize the use of control volumes to maintain a practical problem-solving approach that is also theoretically inclusive.

## ***Proven Problem-Solving Methodology***

The Fox-McDonald solution methodology used in this text is illustrated in numerous examples in each chapter, with each one picked to illustrate an important aspect. Solutions presented in the examples have been prepared to illustrate good solution technique and to explain difficult points of theory. Examples are set apart in format from the text so that they are easy to identify and follow. Additional important information about the text and our procedures is given in “Note to Students” in Section 1.1. We urge you to study this section carefully and to integrate the suggested procedures into your problem-solving and results-presentation approaches.

## ***SI and English Units***

SI units are used in about 70 percent of both example and end-of-chapter problems. English Engineering units are retained in the remaining problems to provide experience with this traditional system and to highlight conversions among unit systems.

## ***Goals and Advantages of Using This Text***

Complete explanations presented in the text, together with numerous detailed examples, make this book understandable for students, freeing the instructor to depart from conventional lecture teaching methods. Classroom time can be used to bring in outside material, expand on special topics (such as non-Newtonian flow, boundary-layer flow, lift and drag, or experimental methods), solve example problems, or explain difficult points of assigned homework problems. Thus, each

class period can be used in the manner most appropriate to meet student needs.

When students finish the fluid mechanics course, we expect them to be able to apply the governing equations to a variety of problems, including those they have not encountered previously. We particularly emphasize physical concepts throughout to help students model the variety of phenomena that occur in real fluid flow situations. Although we collect useful equations at the end of each chapter, we stress that our philosophy is to minimize the use of so-called “magic formulas” and emphasize the systematic and fundamental approach to problem-solving. By following this format, we believe students develop confidence in their ability to apply the material and to find that they can reason out solutions to rather challenging problems.

The book is well suited for independent study by students or practicing engineers. Its readability and clear examples help build confidence. Answers to selected problems are included, so students may check their own work.

## ***Topical Coverage***

The material has been selected carefully to include a broad range of topics suitable for a one- or two-semester course at the junior or senior level. We assume a background in rigid-body dynamics, mathematics through differential equations, and thermodynamics.

The text material is organized into broad topic areas:

- Introductory concepts, scope of fluid mechanics, and fluid statics (Chapters 1, 2, and 3)
- Development and application of control volume forms of basic equations (Chapter 4)
- Development and application of differential forms of basic equations (Chapters 5 and 6)
- Dimensional analysis and correlation of experimental data (Chapter 7)
- Applications for internal viscous incompressible flows (Chapter 8)
- Applications for external viscous incompressible flows (Chapter 9)
- Analysis of fluid machinery and system applications (Chapter 10)

- Analysis and applications of open-channel flows (Chapter 11)
- Analysis and applications of one-dimensional compressible flows (Chapter 12)

Chapter 4 deals with analysis using both finite and differential control volumes. The Bernoulli equation is derived as an example application of the basic equations to a differential control volume. Being able to use the Bernoulli equation in Chapter 4 allows us to include more challenging problems dealing with the momentum equation for finite control volumes.

Another derivation of the Bernoulli equation is presented in Chapter 6, where it is obtained by integrating Euler's equation along a streamline. If an instructor chooses to delay introducing the Bernoulli equation, the challenging problems from Chapter 4 may be assigned during study of Chapter 6.

## Text Features

This edition incorporates a number of features that enhance learning:

- **Learning Objectives:** A set of specific Learning Objectives has been developed for the material in each chapter. These are a set of testable specific skills and knowledge that a student should be able to achieve after completing the material in the chapter. Representative questions designed to test whether the student has, in fact, achieved those skills and knowledge is on the Instructor Companion website.
- **Chapter Summary and Useful Equations:** At the end of each chapter, we summarize the major contributions the material has made to meeting the Learning Objectives. Also, as indicated previously, we provide a list of equations that are most commonly used in problem solving.
- **End-of-Chapter Problems:** Problems in each chapter are arranged by topic and grouped according to the chapter section headings. Within each topic they generally increase in complexity or difficulty. This makes it easy for the instructor to assign homework problems at the appropriate difficulty level for each section of the book.
- **Fluid Mechanics Concept Inventory:** The Fluid Mechanics Concept Inventory (FMCI) was developed under an NSF grant to the Foundation Coalition by faculty at the University of Wisconsin (J. Martin and J. Mitchell) and the University of Illinois (A. Jacobi and T. Newell). The inventory is used to a) evaluate whether students understand fluids concepts as opposed to be able to do calculations, and b) improve the teaching of fluids to correct student misconceptions.
- **Design and Open-ended Problems:** Where appropriate, we have provided open-ended design problems. Students could be assigned to work in teams to solve these problems. These problems encourage students to spend more time exploring applications of fluid mechanics principles to

the design of devices and systems. These design problems are available on the Instructor Companion website for many chapters.

## New to This Edition

This edition has been edited significantly and incorporates a number of changes to previous editions:

- **End of Chapter Problems:** Approximately 5 new problems have been authored for each chapter and roughly 50% of the problems have been revised or updated. The number of problems at the end of each chapter has been significantly reduced and selected to illustrate the important aspects of the material. The end-of-chapter problems that have been removed from the previous edition are available on the Instructor Companion website.
- **Instructor-only Problems:** Approximately 25 % of the problems have been set aside as Instructor-only problems that can be assigned at the discretion of the instructor. These are mostly new problems developed for this edition.
- **Show/Hide Solutions:** Approximately 15 % of the problems in the enhanced ebook feature solutions behind show/hide buttons. This feature will allow students to check the intermediate steps of their work.
- **Case Study:** Each chapter is introduced with a Case Study that illustrates the application of the material in the chapter. Most of the case studies in the previous edition have been updated and replaced with more recent applications.
- **Videos:** For many of the chapter subjects, short videos are available that illustrate a specific phenomenon. These videos, which are available in the enhanced ebook, are indicated by an icon in the margin of the text. We also include references to much more extensive collections of videos on a wide range of fluid mechanics topics.
- **References:** The end-of-chapter references have been updated and edited to give the current references most relevant to the material.
- **Computational Fluid Dynamics (CFD):** The material on CFD has been updated to reflect the current state of the art and moved to an appendix with current references.

### Additional Resources in the Enhanced Ebook:

The following resources are available for students enrolled in classes that use the enhanced ebook.

- **Excel Files:** The Excel files used to solve examples in the text are available on the Student Companion Website. These files can be used to explore the solution further or as a guide to the development of new solutions using Excel.
- **A Brief Review of Microsoft Excel:** This is an online tutorial prepared by Philip Pritchard that will aid students in using Excel to solve the end-of-chapter problems.

- **Supplemental Chapter 12 Content:** This is advanced material relevant to Chapter 12 Introduction to Compressible Fluid Flow.

## Resources for Instructors

In addition to the materials available to students, the following resources are available to instructors who adopt this text on the Instructor Companion Website at [www.wiley.com/go/mitchell/foxfluidmechanics10e](http://www.wiley.com/go/mitchell/foxfluidmechanics10e)

- **Solutions Manual:** The solutions manual contains a detailed solution for all homework problems. The expected solution difficulty is indicated, and each solution is prepared in the same systematic way as the example solutions in the printed text. Each solution begins from governing equations, clearly states assumptions, reduces governing equations to computing equations, obtains an algebraic result, and finally substitutes numerical values to obtain a quantitative answer. Solutions may be reproduced for classroom or library use, eliminating the labor of problem-solving for the instructor.
- **Learning Objective Assessment Questions:** These are questions designed to directly assess whether students have achieved the Learning Objectives of the chapter. These can be used in in-class discussions and assigned homework.
- **Problem Key:** The key provides the correspondence between the problems in this tenth edition and those that were renumbered from the ninth edition.
- **PowerPoint Lecture Slides:** Lecture slides outline the concepts in the book and include appropriate illustrations and equations.
- **Image Gallery:** Illustrations are taken from the text in a format appropriate to include in lecture presentations.
- **Sample Syllabi:** Syllabi appropriate for use in teaching a one-semester course in fluid mechanics are provided. First-time instructors will find these a helpful guide to creating an appropriate emphasis on the different topics.
- **Instructor-only Problems:** These are usually new problems that are available to the instructor only and that can be assigned as new challenges to the students.
- **Design Problems:** A set of open-ended design problems have been developed for appropriate chapters. They are designed to integrate material in the chapter and would

be expected to take one to two weeks of homework time for students working in small teams.

- **Supplemental Chapter 12 Content:** This is the material on Fanno flow, Rayleigh flow, and two-dimensional compressible flow that had been developed for previous editions.

## Acknowledgments

This tenth edition continues the evolution of this classic text to meet the needs of students and instructors in fluid mechanics. It continues the tradition of providing a pedagogically sound introduction to the subject of fluids as created by the original authors, Robert Fox and Alan McDonald. Their focus on fundamentals provides a base for students who take only one course in fluids or who continue their studies. The basic aspects covered give students an introduction to the application of fluid mechanics in practice, and the focus on basic principles gives a solid foundation for further study.

Even though the original authors have not been involved with the later editions, we have tried to preserve their enthusiasm for the subject and their personal insights into fluid behavior. Their comments add a dimension not normally found in textbooks and enhance students' understanding of this important subject.

Over the years, many students and faculty have provided recommendations and insight that have shaped the subsequent editions of this book. The current edition thus contains the input of many instructors and researchers in the fluids field that supplements and supports the approach of the original authors.

It is not possible to acknowledge all of the contributors individually, but their collective efforts have been crucial to the success of this text. Most recently, Philip J. Pritchard, the author of the previous edition, introduced many significant improvements and developed the extensive Excel spreadsheets used for Example solutions. Valuable suggestions for updating the previous edition have been made by Tom Acker, Arindam Banerjee, Mark Cappelli; Eun Jung Chae, Melinda Keller, Christopher Pascual, Philippe Sucusky, Sindy KY Tang, and Pavlos Vlachos. We hope that colleagues and others who use this book continue to provide input, for their contributions are essential to maintaining the quality and relevance of this work.

John W. Mitchell  
March, 2019

# Contents

**OC** Content available in eBook.

**SS** Student solution available in interactive e-text.

## CHAPTER 1 INTRODUCTION 1

- 1.1 Introduction to Fluid Mechanics 2
  - Note to Students 2
  - Scope of Fluid Mechanics 3
  - Definition of a Fluid 3
- 1.2 Basic Equations 4
- 1.3 Methods of Analysis 5
  - System and Control Volume 6
  - Differential versus Integral Approach 7
  - Methods of Description 7
- 1.4 Dimensions and Units 9
  - Systems of Dimensions 9
  - Systems of Units 10
  - Preferred Systems of Units 11
  - Dimensional Consistency and “Engineering” Equations 11
- 1.5 Analysis of Experimental Error 13
- 1.6 Summary 14
- References 14

## CHAPTER 2 FUNDAMENTAL CONCEPTS 15

- 2.1 Fluid as a Continuum 16
- 2.2 Velocity Field 17
  - One-, Two-, and Three-Dimensional Flows 18
  - Timelines, Pathlines, Streaklines, and Streamlines 19
- 2.3 Stress Field 23
- 2.4 Viscosity 25
  - Newtonian Fluid 26
  - Non-Newtonian Fluids 28
- 2.5 Surface Tension 29
- 2.6 Description and Classification of Fluid Motions 30
  - Viscous and Inviscid Flows 32
  - Laminar and Turbulent Flows 34
  - Compressible and Incompressible Flows 34
  - Internal and External Flows 35

- 2.7 Summary and Useful Equations 36
- References 37

## CHAPTER 3 FLUID STATICS 38

- 3.1 The Basic Equation of Fluid Statics 39
- 3.2 The Standard Atmosphere 42
- 3.3 Pressure Variation in a Static Fluid 43
  - Incompressible Liquids: Manometers 43
  - Gases 48
- 3.4 Hydrostatic Force on Submerged Surfaces 50
  - Hydrostatic Force on a Plane Submerged Surface 50
  - Hydrostatic Force on a Curved Submerged Surface 57
- 3.5 Buoyancy and Stability 60
- 3.6 Fluids in Rigid-Body Motion 63
- 3.7 Summary and Useful Equations 68
- References 69

## CHAPTER 4 BASIC EQUATIONS IN INTEGRAL FORM FOR A CONTROL VOLUME 70

- 4.1 Basic Laws for a System 71
  - Conservation of Mass 71
  - Newton’s Second Law 72
  - The Angular-Momentum Principle 72
  - The First Law of Thermodynamics 72
  - The Second Law of Thermodynamics 73
- 4.2 Relation of System Derivatives to the Control Volume Formulation 73
  - Derivation 74
  - Physical Interpretation 76
- 4.3 Conservation of Mass 77
  - Special Cases 78
- 4.4 Momentum Equation for Inertial Control Volume 82
  - Differential Control Volume Analysis 93



Control Volume Moving with Constant Velocity	97	Static, Stagnation, and Dynamic Pressures	169
4.5 Momentum Equation for Control Volume with Rectilinear Acceleration	99	Applications	171
4.6 Momentum Equation for Control Volume with Arbitrary Acceleration	105	Cautions on Use of the Bernoulli Equation	176
4.7 The Angular-Momentum Principle	110	6.3 The Bernoulli Equation Interpreted as an Energy Equation	177
Equation for Fixed Control Volume	110	6.4 Energy Grade Line and Hydraulic Grade Line	181
Equation for Rotating Control Volume	114	6.5 Unsteady Bernoulli Equation: Integration of Euler's Equation Along a Streamline	183
4.8 The First and Second Laws of Thermodynamics	118	6.6 Irrotational Flow	185
Rate of Work Done by a Control Volume	119	Bernoulli Equation Applied to Irrotational Flow	185
Control Volume Equation	121	Velocity Potential	186
4.9 Summary and Useful Equations	125	Stream Function and Velocity Potential for Two-Dimensional, Irrotational, Incompressible Flow: Laplace's Equation	187
<b>CHAPTER 5 INTRODUCTION TO DIFFERENTIAL ANALYSIS OF FLUID MOTION</b>	128	Elementary Plane Flows	189
5.1 Conservation of Mass	129	Superposition of Elementary Plane Flows	191
Rectangular Coordinate System	129	6.7 Summary and Useful Equations	200
Cylindrical Coordinate System	133	References	201
5.2 Stream Function for Two-Dimensional Incompressible Flow	135	<b>CHAPTER 7 DIMENSIONAL ANALYSIS AND SIMILITUDE</b>	202
5.3 Motion of a Fluid Particle (Kinematics)	137	7.1 Nondimensionalizing the Basic Differential Equations	204
Fluid Translation: Acceleration of a Fluid Particle in a Velocity Field	138	7.2 Buckingham Pi Theorem	206
Fluid Rotation	144	7.3 Significant Dimensionless Groups in Fluid Mechanics	212
Fluid Deformation	147	7.4 Flow Similarity and Model Studies	214
5.4 Momentum Equation	151	Incomplete Similarity	216
Forces Acting on a Fluid Particle	151	Scaling with Multiple Dependent Parameters	221
Differential Momentum Equation	152	Comments on Model Testing	224
Newtonian Fluid: Navier–Stokes Equations	152	7.5 Summary and Useful Equations	225
5.5 Summary and Useful Equations	160	References	226
References	161	<b>CHAPTER 8 INTERNAL INCOMPRESSIBLE VISCOUS FLOW</b>	227
<b>CHAPTER 6 INCOMPRESSIBLE INVISCID FLOW</b>	162	8.1 Internal Flow Characteristics	228
6.1 Momentum Equation for Frictionless Flow: Euler's Equation	163	Laminar versus Turbulent Flow	228
6.2 Bernoulli Equation: Integration of Euler's Equation Along a Streamline for Steady Flow	167	The Entrance Region	229
Derivation Using Streamline Coordinates	167	PART A. FULLY DEVELOPED LAMINAR FLOW	230
Derivation Using Rectangular Coordinates	168	8.2 Fully Developed Laminar Flow Between Infinite Parallel Plates	230
		Both Plates Stationary	230

Upper Plate Moving with Constant Speed, $U$	236
8.3 Fully Developed Laminar Flow in a Pipe	241
<b>PART B. FLOW IN PIPES AND DUCTS</b>	245
8.4 Shear Stress Distribution in Fully Developed Pipe Flow	246
8.5 Turbulent Velocity Profiles in Fully Developed Pipe Flow	247
8.6 Energy Considerations in Pipe Flow	251
Kinetic Energy Coefficient	252
Head Loss	252
8.7 Calculation of Head Loss	253
Major Losses: Friction Factor	253
Minor Losses	258
Pumps, Fans, and Blowers in Fluid Systems	262
Noncircular Ducts	262
8.8 Solution of Pipe Flow Problems	263
Single-Path Systems	264
Multiple-Path Systems	276
<b>PART C. FLOW MEASUREMENT</b>	279
8.9 Restriction Flow Meters for Internal Flows	279
The Orifice Plate	282
The Flow Nozzle	286
The Venturi	286
The Laminar Flow Element	287
Linear Flow Meters	288
Traversing Methods	289
8.10 Summary and Useful Equations	290
References	292
<b>CHAPTER 9 EXTERNAL INCOMPRESSIBLE VISCOUS FLOW</b>	293
<b>PART A. BOUNDARY LAYERS</b>	295
9.1 The Boundary Layer Concept	295
9.2 Laminar Flat Plate Boundary Layer: Exact Solution	299
9.3 Momentum Integral Equation	302
9.4 Use of the Momentum Integral Equation for Flow with Zero Pressure Gradient	306
Laminar Flow	307
Turbulent Flow	311
9.5 Pressure Gradients in Boundary Layer Flow	314

<b>PART B. FLUID FLOW ABOUT IMMERSED BODIES</b>	316
9.6 Drag	316
Pure Friction Drag: Flow over a Flat Plate Parallel to the Flow	317
Pure Pressure Drag: Flow over a Flat Plate Normal to the Flow	320
Friction and Pressure Drag: Flow over a Sphere and Cylinder	320
Streamlining	326
9.7 Lift	328
9.8 Summary and Useful Equations	340
References	342

## **CHAPTER 10 FLUID MACHINERY** 343

10.1 Introduction and Classification of Fluid Machines	344
Machines for Doing Work on a Fluid	344
Machines for Extracting Work (Power) from a Fluid	346
Scope of Coverage	348
10.2 Turbomachinery Analysis	348
The Angular Momentum Principle: The Euler Turbomachine Equation	348
Velocity Diagrams	350
Performance—Hydraulic Power	352
Dimensional Analysis and Specific Speed	353
10.3 Pumps, Fans, and Blowers	358
Application of Euler Turbomachine Equation to Centrifugal Pumps	358
Application of the Euler Equation to Axial Flow Pumps and Fans	359
Performance Characteristics	362
Similarity Rules	367
Cavitation and Net Positive Suction Head	371
Pump Selection: Applications to Fluid Systems	374
Blowers and Fans	380
10.4 Positive Displacement Pumps	384
10.5 Hydraulic Turbines	387
Hydraulic Turbine Theory	387
Performance Characteristics for Hydraulic Turbines	389
10.6 Propellers and Wind Turbines	395
Propellers	395
Wind Turbines	400

10.7 Compressible Flow Turbomachines 406  
 Application of the Energy Equation to a  
 Compressible Flow Machine 406  
 Compressors 407  
 Compressible-Flow Turbines 410  
 10.8 Summary and Useful Equations 410  
 References 412

**CHAPTER 11 FLOW IN OPEN CHANNELS 414**

11.1 Basic Concepts and Definitions 416  
 Simplifying Assumptions 416  
 Channel Geometry 418  
 Speed of Surface Waves and the Froude  
 Number 419  
 11.2 Energy Equation for Open-Channel Flows 423  
 Specific Energy 425  
 Critical Depth: Minimum Specific Energy 426  
 11.3 Localized Effect of Area Change  
 (Frictionless Flow) 431  
 Flow over a Bump 431  
 11.4 The Hydraulic Jump 435  
 Depth Increase Across a Hydraulic Jump 438  
 Head Loss Across a Hydraulic Jump 439  
 11.5 Steady Uniform Flow 441  
 The Manning Equation for Uniform Flow 443  
 Energy Equation for Uniform Flow 448  
 Optimum Channel Cross Section 450  
 11.6 Flow with Gradually Varying Depth 451  
 Calculation of Surface Profiles 452  
 11.7 Discharge Measurement Using Weirs 455  
 Suppressed Rectangular Weir 455  
 Contracted Rectangular Weirs 456  
 Triangular Weir 456  
 Broad-Crested Weir 457  
 11.8 Summary and Useful Equations 458  
 References 459

**CHAPTER 12 INTRODUCTION TO COMPRESSIBLE FLOW 460**

12.1 Review of Thermodynamics 461  
 12.2 Propagation of Sound Waves 467  
 Speed of Sound 467  
 Types of Flow—The Mach Cone 471  
 12.3 Reference State: Local Isentropic Stagnation  
 Properties 473

Local Isentropic Stagnation Properties for the  
 Flow of an Ideal Gas 474  
 12.4 Critical Conditions 480  
 12.5 Basic Equations for One-Dimensional  
 Compressible Flow 480  
 Continuity Equation 481  
 Momentum Equation 481  
 First Law of Thermodynamics 481  
 Second Law of Thermodynamics 482  
 Equation of State 483  
 12.6 Isentropic Flow of an Ideal Gas: Area  
 Variation 483  
 Subsonic Flow,  $M < 1$  485  
 Supersonic Flow,  $M > 1$  486  
 Sonic Flow,  $M = 1$  486  
 Reference Stagnation and Critical Conditions  
 for Isentropic Flow of an Ideal Gas 487  
 Isentropic Flow in a Converging Nozzle 492  
 Isentropic Flow in a Converging-Diverging  
 Nozzle 496  
 12.7 Normal Shocks 501  
 Basic Equations for a Normal Shock 501  
 Normal-Shock Flow Functions for  
 One-Dimensional Flow of an Ideal Gas 503  
 12.8 Supersonic Channel Flow with Shocks 507  
 12.9 Summary and Useful Equations 509  
 References 511

**OC** Problems (Available in e-text for students) P-1

**APPENDIX A FLUID PROPERTY DATA A-1**

**APPENDIX B VIDEOS FOR FLUID MECHANICS A-13**

**APPENDIX C SELECTED PERFORMANCE CURVES FOR PUMPS AND FANS A-15**

**APPENDIX D FLOW FUNCTIONS FOR COMPUTATION OF COMPRESSIBLE FLOW A-26**

**APPENDIX E ANALYSIS OF EXPERIMENTAL UNCERTAINTY A-29**

**APPENDIX F INTRODUCTION TO COMPUTATIONAL FLUID DYNAMICS A-35**

Index I-1



## Chapter 1 Problems

**SS** Student solution available in interactive e-text.

### Definition of a Fluid: Basic Equations

1.1 Describe the conditions for which the following substances can be considered liquids.

Tar	Honey	Wax	Propane
Carbon dioxide	Sea water	Sand	Toothpaste

1.2 Give a word statement of each of the five basic conservation laws stated in Section 1.2 as they apply to a system.

### Methods of Analysis

1.3 The barrel of a bicycle tire pump becomes quite warm during use. Explain the mechanisms responsible for the temperature increase.

**SS** 1.4 In a pollution control experiment, minute solid particles (typical mass  $1 \times 10^{-13}$  slug) are dropped in air. The terminal speed of the particles is measured to be 0.2 ft/s. The drag of these particles is given by  $F_D = kV$ , where  $V$  is the instantaneous particle speed. Find the value of the constant  $k$ . Find the time required to reach 99 percent of terminal speed.

1.5 A rocket payload with a weight on earth of 2000 lbf is sent to the moon. The acceleration due to gravity in the moon is  $1/6^{\text{th}}$  that of the earth. Determine the mass of the payload on the earth and on the moon and the weight of the payload on the moon in SI, BG, EE units.

1.6 The English perfected the longbow as a weapon after the Medieval period. In the hands of a skilled archer, the longbow was reputed to be accurate at ranges to 100 m or more. If the maximum altitude of an arrow is less than  $h = 10$  m while traveling to a target 100 m away from the archer, and neglecting air resistance, estimate the speed and angle at which the arrow must leave the bow. Plot the required release speed and angle as a function of height  $h$ .

1.7 Air at standard atmospheric conditions enters the 6 in. diameter inlet of an air compressor at a velocity of 20 ft/s. The air is compressed and leaves the compressor through a 6 in. diameter outlet at 80 psia and 150°F. Determine the mass flow rate of the air and the exit velocity.

1.8 A water flow of 4.5 slug/s at 60°F enters the condenser of steam turbine and leaves at 140°F. Determine the heat transfer rate (Btu/hr).

1.9 Determine the weight (N) and specific volume of a cubic meter of air at 101 kPa and 15°C. Determine the specific volume if the air is cooled to  $-10^\circ\text{C}$  at constant pressure.

**SS** 1.10 Determine the specific weight, specific volume, and density of air at 40°F and 50 psia in BG units. Determine the specific weight, specific volume, and density when the air is then compressed isentropically to 100 psia.

### Dimensions and Units

1.11 For each quantity listed, indicate dimensions using mass as a primary dimension, and give typical SI and English units:

- (a) Power
- (b) Pressure
- (c) Modulus of elasticity

- (d) Angular velocity
- (e) Energy
- (f) Moment of a force
- (g) Momentum
- (h) Shear stress
- (i) Strain
- (j) Angular momentum

1.12 For each quantity listed, indicate dimensions using force as a primary dimension, and give typical SI and English units:

- (a) Power
- (b) Pressure
- (c) Modulus of elasticity
- (d) Angular velocity
- (e) Energy
- (f) Momentum
- (g) Shear stress
- (h) Specific heat
- (i) Thermal expansion coefficient
- (j) Angular momentum

1.13 The maximum theoretical flow rate (slug/s) for air flow through a supersonic nozzle is given as

$$\dot{m}_{\max} = 2.38 \frac{A_t p_0}{\sqrt{T_0}}$$

where  $A_t$  is the nozzle throat area ( $\text{ft}^2$ ),  $p_0$  is the supply tank pressure (psia), and  $T_0$  is the air temperature in the tank ( $^\circ\text{R}$ ). Determine the dimensions and units of the constant 2.38. Determine the equivalent equation in SI units.

1.14 The mean free path  $\lambda$  of a molecule of gas is the average distance it travels before collision with another molecule. It is given by

$$\lambda = C \frac{m}{\rho d^2}$$

where  $m$  and  $d$  are the molecule's mass and diameter, respectively, and  $\rho$  is the gas density. Determine the dimensions of constant  $C$  for a dimensionally consistent equation.

1.15 The density of a sample of sea water is 1.99 slug/ft<sup>3</sup>. Determine the value of density in SI and EE units, and the value of specific weight in SI, BG and EE units.

1.16 A fluid occupying 3.2 m<sup>3</sup> has a mass of 4 mg. Calculate its density and specific volume in SI, EE, and BG units.

1.17 Derive the following conversion factors:

- (a) Convert a pressure of 1 psi to kPa.
- (b) Convert a volume of 1 liter to gallons.
- (c) Convert a viscosity of 1 lbf·s/ft<sup>2</sup> to N·s/m<sup>2</sup>.

1.18 Express the following in SI units:

- (a) 100 cfm (ft<sup>3</sup>/min)
- (b) 5 gal
- (c) 65 mph
- (d) 5.4 acres

## P-2 Chapter 1 Problems

1.19 Express the following in BG units:

- (a) 50 m<sup>2</sup>
- (b) 250 cc
- (c) 100 kW
- (d) 5 kg/m<sup>2</sup>

1.20 Derive the conversion factors for the following quantities for volume flow rate

- (a) Converting in<sup>3</sup>/min to mm<sup>3</sup>/s.
- (b) Converting gallons per minute (gpm) to m<sup>3</sup>/s.
- (c) Converting gpm to liters/min.
- (d) Converting cubic feet per minute (cfm) to m<sup>3</sup>/s.

### Analysis of Experimental Error

1.21 Calculate the density of standard air in a laboratory from the ideal gas equation of state. Estimate the experimental uncertainty in the air density calculated for standard conditions (29.9 in. of mercury and 59°F) if the uncertainty in measuring the barometer height is ±0.1 in. of mercury and the uncertainty in measuring temperature is ±0.5°F. (Note that 29.9 in. of mercury corresponds to 14.7 psia.)

1.22 A parameter that is often used in describing pump performance is the specific speed,  $N_{S_{cu}}$ , given by

$$N_{S_{cu}} = \frac{N(\text{rpm})[Q(\text{gpm})]^{1/2}}{[H(\text{ft})]^{3/4}}$$

Determine the units of specific speed. For a pump with a specific speed of 200, determine the specific speed in SI units with angular velocity in rad/s.

1.23 The mass flow rate in a water flow system determined by collecting the discharge over a timed interval is 0.2 kg/s. The scales used can be read to the nearest 0.05 kg and the stopwatch is accurate to 0.2 s. Estimate the precision with which the flow rate can be calculated for time intervals of (a) 10 s and (b) 1 min. **SS**

1.24 From Appendix A, the viscosity  $\mu(\text{N}\cdot\text{s}/\text{m}^2)$  of water at temperature  $T(\text{K})$  can be computed from  $\mu = A10^{B/(T-C)}$ , where  $A = 2.414 \times 10^{-5} \text{ N}\cdot\text{s}/\text{m}^2$ ,  $B = 247.8 \text{ K}$ , and  $C = 140 \text{ K}$ . Determine the viscosity of water at 30°C, and estimate its uncertainty if the uncertainty in temperature measurement is ±0.5°C.

1.25 The height of a building may be estimated by measuring the horizontal distance to a point on the ground and the angle from this point to the top of the building. Assuming that these measurements are  $L = 100 \pm 0.5 \text{ ft}$  and  $\theta = 30 \pm 0.2^\circ$ , estimate the height  $H$  of the building and the uncertainty in the estimate. For the same building height and measurement uncertainties, use *Excel's Solver* to determine the angle (and the corresponding distance from the building) at which measurements should be made to minimize the uncertainty in estimated height. Evaluate and plot the optimum measurement angle as a function of building height for  $50 \leq H \leq 1000 \text{ ft}$ .

1.26 An American golf ball has a mass of  $1.62 \pm 0.01 \text{ oz}$  and a nominal diameter of 1.68 in. Determine the precision that the diameter of the ball must be measured so that the uncertainty of the density of the ball is ±1 percent.

# CHAPTER 1

## Introduction

- 1.1 Introduction to Fluid Mechanics
- 1.2 Basic Equations
- 1.3 Methods of Analysis

- 1.4 Dimensions and Units
- 1.5 Analysis of Experimental Error
- 1.6 Summary

### Case Study

Wind generated electricity is increasingly becoming a major factor in meeting U.S. energy needs. In 2017, wind energy provided  $254 \times 10^9$  kWh of electricity, or about 6.3 percent of the total electrical use (EIA, [www.eia.gov/](http://www.eia.gov/)). In several states, over 10 percent of the electricity generated came from wind. The total production has increased by 30 percent over that of 2015, showing the rapid growth of wind power.

Electricity from the wind is produced in wind turbine farms, as shown in the figure. The individual turbines are large, with the height of the hub reaching to 70 m (220 ft) and the blades up to 35 m (120 ft) in diameter. The total weight of the blades, nacelle, and tower is about 165 tons. The nacelle contains the gearbox and the generator, with the transmission lines to the grid in the tower. A turbine this size is rated at 1.5 MW, which is enough electricity to supply about 250 U.S. homes.

The kinetic energy of the wind is the source of power for a wind turbine. Using the conservation of mass relation, the maximum efficiency of a wind turbine has been established at 59.3 percent. This maximum is known as Betz's law after the German physicist Albert Betz who derived the relation in 1919. Practical utility-scale wind turbines achieve 75–80 percent of the maximum, thus extracting about 45 percent of the available energy of the wind.

The design of turbine blades has evolved from the flat surfaces used on windmills to the propeller shapes seen on modern turbines. The pitch of the blade, which is the angle between the blade and the oncoming wind, is increased as the wind speed

increases. This is done to provide a match between the wind and the blade surface so that the wind flows smoothly over the blade. Because the velocity of the blade increases from the hub to the tip, the blade shape twists accordingly to maintain a constant pitch along the blade. Such designs allow the high efficiency attained in wind turbines. The basic fluid mechanics that we will study in this text provides the basis for the aerodynamic design of devices such as wind turbines.



Dale/Adobe Stock Photo

**Wind turbine farm.**

## Learning Objectives

After completing this chapter, you should be able to

- Explain the definition of a fluid in physical terms.
- State and explain the basic laws of fluid mechanics.
- Define “system” and “control volume” and explain the difference.
- Describe the dimensions and units of the systems used in fluid mechanics.
- Estimate the uncertainty in a physical measurement.

### 1.1 Introduction to Fluid Mechanics

We decided to title this textbook “Introduction to ...” for the following reason: After studying the text, you will *not* be able to design the streamlining of a new car or an airplane, or design a new heart valve, or select the correct air extractors and ducting for a \$100 million building; however, you *will* have developed a good understanding of the concepts behind all of these, and many other applications, and have made significant progress toward being ready to work on such state-of-the-art fluid mechanics projects.

To start toward this goal, in this chapter we cover some very basic topics: a case study, what fluid mechanics encompasses, the standard engineering definition of a fluid, and the basic equations and methods of analysis. Finally, we discuss some common engineering student pitfalls in areas such as unit systems and experimental analysis.

#### Note to Students

This is a student-oriented book: We believe it is quite comprehensive for an introductory text, and a student can successfully self-teach from it. However, most students will use the text in conjunction with one or two undergraduate courses. In either case, we recommend a thorough reading of the relevant chapters. In fact, a good approach is to read a chapter quickly once, then reread more carefully a second and even a third time, so that concepts develop a context and meaning. While students often find fluid mechanics quite challenging, we believe this approach, supplemented by your instructor’s lectures that will hopefully amplify and expand upon the text material, will show fluid mechanics to be a fascinating and varied field of study.

There are some prerequisites for this text. We assume you have already studied introductory thermodynamics, as well as statics, dynamics, and calculus; however, as needed, we will review some of this material.

It is our strong belief that one learns best by *doing*. This is true whether the subject under study is fluid mechanics, or soccer. The fundamentals in any of these are few, and mastery of them comes through practice. *Thus it is extremely important that you solve problems.* The numerous problems included at the end of each chapter provide the opportunity to practice applying fundamentals to the solution of problems. Even though we provide for your convenience a summary of useful equations at the end of each chapter, you should avoid the temptation to adopt a so-called plug-and-chug approach to solving problems. Most of the problems are such that this approach simply will not work. In solving problems, we strongly recommend that you proceed using the following logical steps:

- 1 State briefly and concisely in your own words the information given.
- 2 State the information to be found.
- 3 Draw a schematic of the system or control volume to be used in the analysis. Be sure to label the boundaries of the system or control volume and label appropriate coordinate directions.
- 4 Give the appropriate mathematical formulation of the *basic* laws that you consider necessary to solve the problem.
- 5 List the simplifying assumptions that you feel are appropriate in the problem.



- 6 Complete the analysis algebraically before substituting numerical values. This is especially important if you are using software to solve the problem.
- 7 Substitute numerical values to obtain a numerical answer.
  - (a) Reference the source of values for any physical properties.
  - (b) Be sure the significant figures in the answer are consistent with the given data.
  - (c) Check the units of each term to be certain they are consistent.
- 8 Check the answer and review the assumptions made in the solution to make sure they are reasonable.
- 9 Label the answer.

In your initial work this problem format may seem unnecessary and even long-winded. However, it is our experience that this approach to problem solving is ultimately the most efficient. It will also prepare you to be a successful professional, for which a major prerequisite is to be able to communicate information and the results of an analysis clearly and precisely. *This format is used in all examples presented in this text*; answers to examples are rounded to three significant figures.

The problems at the end of each chapter range in degree of difficulty. For many, pencil and paper together with a calculator will suffice. This is especially true for those problems we have designed to illustrate a single principle or concept. However, there are many more complex problems, and we have found that using software tools is a more appropriate and satisfactory approach to obtaining a solution. We have provided an *Excel* tutorial and solutions for many of the book's examples on the website that can be used to help you get started with this tool. Additionally, there are a large number of other equation solvers that students have found very useful, including *EES*, *MATLAB*, and *Mathematica*. We encourage you to learn to use one of these tools as virtually all problems you will encounter in practice are complicated.

## Scope of Fluid Mechanics

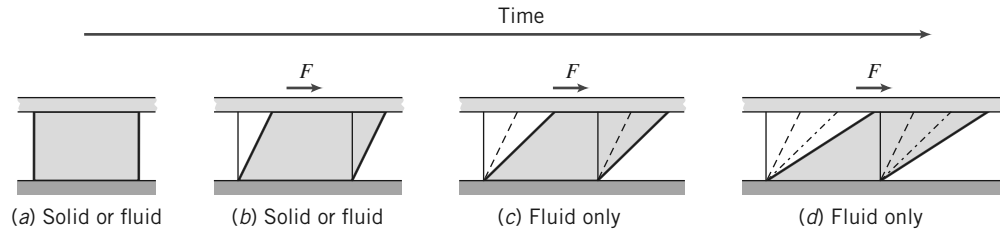
As the name implies, fluid mechanics is the study of fluids at rest or in motion. The subject has applications to a wide range of traditional subjects such as the design of dam systems, water delivery systems, pumps and compressors, and the aerodynamics of automobiles and airplanes. Fluid mechanics has facilitated the development of new technology in the environmental and energy area such as large-scale wind turbines and oil spill cleanups. Medical advances in the understanding and treatment of flow problems in the circulatory and respiratory system have been aided by fluid mechanics applications. The modeling of atmospheric circulation and ocean currents that aids understanding of climate change is based on fluid mechanics principles. Possibly the greatest advance in fluid mechanics in recent years is the ability to model extremely complex flows using software. The technique known as computational fluid dynamics (CFD) has at its heart the basic relations of fluid mechanics.

These are just a small sampling of the newer areas of fluid mechanics, but they illustrate how the discipline is still highly relevant, and increasingly diverse, even though it may be thousands of years old.

## Definition of a Fluid

We are certain that you have a common-sense idea of what a fluid is, as opposed to a solid. Fluids tend to flow when we interact with them whereas solids tend to deform or bend. Engineers need a more formal and precise definition of a fluid: A *fluid* is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be. Because the fluid motion continues under the application of a shear stress, we can also define a fluid as any substance that cannot sustain a shear stress when at rest.

Hence liquids and gases (or vapors) are the forms, or phases, that fluids can take. We wish to distinguish these phases from the solid phase of matter. We can see the difference between solid and fluid behavior in Fig. 1.1. If we place a specimen of either substance between two plates (Fig. 1.1*a*) and then apply a shearing force  $F$ , each will initially deform (Fig. 1.1*b*); however, whereas a solid will then be at rest (assuming the force is not large enough to go beyond its elastic limit), a fluid will *continue* to deform (Fig. 1.1*c, d*, etc.) as long as the force is applied. Note that a fluid in contact with a solid surface



**Fig. 1.1** Difference in behavior of a solid and a fluid due to a shear force.

does not slip. It has the same velocity as that surface because of the *no-slip* condition, which is a very important experimental fact.

The amount of deformation of the solid depends on the solid's modulus of rigidity. In Chapter 2 we will learn that the *rate of deformation* of the fluid depends on the fluid's viscosity  $\mu$ . We refer to solids as being *elastic* and fluids as being *viscous*. More informally, we say that solids exhibit "springiness." For example, when you drive over a pothole, the car bounces up and down due to the metal coil springs compressing and expanding. On the other hand, fluids exhibit friction effects so that the shock absorbers, which contain a fluid that is forced through a small opening, dissipate energy due to the fluid friction and stop the bouncing after a few oscillations.

The idea that substances can be categorized as being either a solid or a liquid holds for most substances, but a number of substances exhibit both springiness and friction. They are termed *viscoelastic*. Many biological tissues are viscoelastic. For example, the synovial fluid in human knee joints lubricates those joints but also absorbs some of the shock occurring during walking or running. Other examples of viscoelastic materials are some polymers, metals at very high temperatures, and bitumen material such as asphalt.

## 1.2 Basic Equations

Analysis of any problem in fluid mechanics necessarily includes statement of the basic laws governing the fluid motion. The basic laws, which are applicable to any fluid, are:

- 1 The conservation of mass
- 2 Newton's second law of motion (also termed the principle of linear momentum)
- 3 The principle of angular momentum
- 4 The first law of thermodynamics
- 5 The second law of thermodynamics

All basic laws are usually not required to solve any one problem. On the other hand, in many problems it is necessary to bring into the analysis additional relations that describe the behavior of physical properties of fluids under given conditions. For example, the *ideal gas* equation of state

$$p = \rho RT \quad (1.1)$$

is a model that relates density to pressure and temperature for many gases under normal conditions. In Eq. 1.1,  $R$  is the gas constant. Values of  $R$  are given in Appendix A for several common gases;  $p$  and  $T$  in Eq. 1.1 are the absolute pressure and absolute temperature, respectively and  $\rho$  is density (mass per unit volume). Example 1.1 illustrates use of the ideal gas equation of state.

It is obvious that the basic laws with which we shall deal are the same as those used in mechanics and thermodynamics. Our task will be to formulate these laws in suitable forms to solve fluid flow problems and to apply them to a wide variety of situations.

### Example 1.1 FIRST LAW APPLICATION TO CLOSED SYSTEM

A piston-cylinder device contains 0.95 kg of oxygen initially at a temperature of 27°C and a pressure due to the weight of 150 kPa (abs). Heat is added to the gas until it reaches a temperature of 627°C. Determine the amount of heat added during the process.

**Given:** Piston-cylinder containing O<sub>2</sub>,  $m = 0.95$  kg.

$$T_1 = 27^\circ\text{C} \quad T_2 = 627^\circ\text{C}$$

**Find:**  $Q_{1 \rightarrow 2}$ .

**Solution:**  $p = \text{constant} = 150$  kPa (abs)

We are dealing with a system,  $m = 0.95$  kg.

**Governing equation:** First law for the system,  $Q_{12} - W_{12} = E_2 - E_1$

**Assumptions:** 1  $E = U$ , since the system is stationary.

2 Ideal gas with constant specific heats.

Under the above assumptions,

$$E_2 - E_1 = U_2 - U_1 = m(u_2 - u_1) = mc_v(T_2 - T_1)$$

The work done during the process is moving boundary work

$$W_{12} = \int_{V_1}^{V_2} p dV = p(V_2 - V_1)$$

For an ideal gas,  $pV = mRT$ . Hence  $W_{12} = mR(T_2 - T_1)$ . Then from the first law equation,

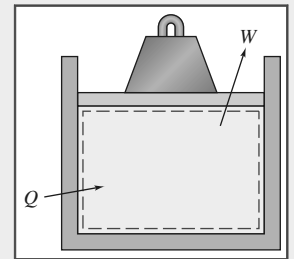
$$Q_{12} = E_2 - E_1 + W_{12} = mc_v(T_2 - T_1) + mR(T_2 - T_1)$$

$$Q_{12} = m(T_2 - T_1)(c_v + R)$$

$$Q_{12} = mc_p(T_2 - T_1) \quad \{R = c_p - c_v\}$$

From Table A.6 in Appendix A, for O<sub>2</sub>,  $c_p = 909.4 \text{ J}/(\text{kg} \cdot \text{K})$ . Solving for  $Q_{12}$ , we obtain

$$Q_{12} = 0.95 \text{ kg} \times 909 \frac{\text{J}}{\text{kg} \cdot \text{K}} \times 600 \text{ K} = 518 \text{ kJ} \leftarrow \frac{Q_{12}}$$



*This problem:*

- Was solved using the nine logical steps discussed earlier.
- Reviewed the use of the ideal gas equation and the first law of thermodynamics for a system.

## 1.3 Methods of Analysis

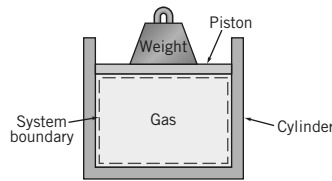
The first step in solving a problem is to define the system that you are attempting to analyze. In basic mechanics, we made extensive use of the *free-body diagram*. In fluid mechanics, we will use a *system* or a *control volume*, depending on the problem being studied. These concepts are identical to the ones you used in thermodynamics (also termed *closed system* and *open system*, respectively). We can use either one to get mathematical expressions for each of the basic laws. In thermodynamics we applied the conservation of mass and the first and second laws of thermodynamics in most problems. In our study of fluid mechanics, we will usually apply conservation of mass and Newton's second law of motion. In thermodynamics our focus was energy; in fluid mechanics it will mainly be forces and motion. We must always be aware of whether we are using a system or a control volume approach because each leads to

different mathematical expressions of these laws. At this point we review the definitions of systems and control volumes.

## System and Control Volume

A *system* is defined as a fixed, identifiable quantity of mass with the system boundaries separating the system from the surroundings. The boundaries of the system may be fixed or movable; however, no mass crosses the system boundaries.

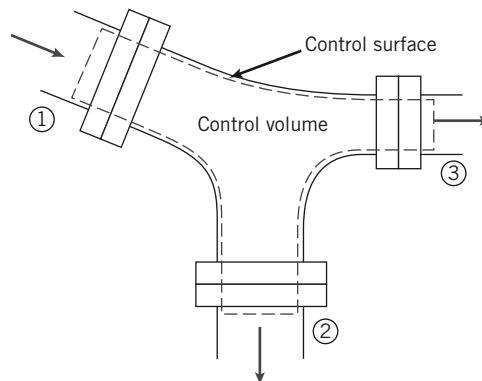
In the familiar piston-cylinder assembly from thermodynamics, Fig. 1.2, the gas in the cylinder is the system. If the gas is heated, the piston will lift the weight; the boundary of the system thus moves. Heat and work may cross the boundaries of the system, but the quantity of matter within the system boundaries remains fixed. No mass crosses the system boundaries.



**Fig. 1.2** Piston-cylinder assembly.

In solid body mechanics courses you used the free-body diagram (system approach) extensively. This was logical because you were dealing with an easily identifiable rigid body. However, in fluid mechanics we normally are concerned with the flow of fluids through devices such as compressors, turbines, pipelines, and nozzles. In these cases it is difficult to focus attention on a fixed identifiable quantity of mass. It is much more convenient, to focus attention on a volume in space through which the fluid flows. Consequently, we use the control volume approach.

A *control volume* is an arbitrary volume in space through which fluid flows. The geometric boundary of the control volume is called the control surface. The control surface may be real or imaginary; it may be at rest or in motion. Figure 1.3 shows flow through a pipe junction, with a control surface drawn on it. Note that some regions of the surface correspond to physical boundaries (the walls of the pipe) and others (at locations ①, ②, and ③) are parts of the surface that are imaginary. For the control volume defined by this surface, we could write equations for the basic laws and obtain results such as the flow rate at outlet ③ given the flow rates at inlet ① and outlet ② or the force required to hold the junction in place. Example 1.2 illustrates how we use a control volume to determine the mass flow rate in a section of a pipe. It is always important to take care in selecting a control volume, as the choice has a big effect on the mathematical form of the basic laws.



**Fig. 1.3** Fluid flow through a pipe junction.

### Example 1.2 MASS CONSERVATION APPLIED TO CONTROL VOLUME

A reducing water pipe section has an inlet diameter of 50 mm and exit diameter of 30 mm. If the steady inlet speed (averaged across the inlet area) is 2.5 m/s, find the exit speed.

**Given:** Pipe, inlet  $D_i = 50$  mm, exit  $D_e = 30$  mm.  
Inlet speed,  $V_i = 2.5$  m/s.

**Find:** Exit speed,  $V_e$ .

**Solution:**

**Assumption:** Water is incompressible (density  $\rho = \text{constant}$ ).

The physical law we use here is the conservation of mass, which you learned in thermodynamics when studying turbines, boilers, and so on. You may have seen mass flow at an inlet or outlet expressed as either  $\dot{m} = VA/v$  or  $\dot{m} = \rho VA$  where  $V$ ,  $A$ ,  $v$  and  $\rho$ , are the speed, area, specific volume, and density, respectively. We will use the density form of the equation.

Hence the mass flow is:

$$\dot{m} = \rho VA$$

Applying mass conservation,

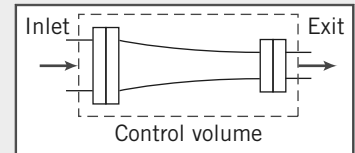
$$\rho V_i A_i = \rho V_e A_e$$

Note:  $\rho_i = \rho_e = \rho$  by our first assumption.

Solving for  $V_e$ ,

$$V_e = V_i \frac{A_i}{A_e} = V_i \frac{\pi D_i^2 / 4}{\pi D_e^2 / 4} = V_i \left( \frac{D_i}{D_e} \right)^2$$

$$V_e = 2.7 \frac{\text{m}}{\text{s}} \left( \frac{50}{30} \right)^2 = 7.5 \frac{\text{m}}{\text{s}} \leftarrow V_e$$



*This problem:*

- Was solved using the nine logical steps.
- Demonstrated the use of a control volume and the mass conservation law.

## Differential versus Integral Approach

The basic laws that we apply in our study of fluid mechanics can be formulated in terms of systems or control volumes. As you might suspect, the equations will look different in the two cases. Both approaches are important in the study of fluid mechanics and both will be developed in the course of our work.

In the case of *infinitesimal* systems the resulting equations are differential equations. Solution of the differential equations of motion provides a means of determining the detailed behavior of the flow. An example might be the pressure distribution on a wing surface.

Frequently the information sought does not require a detailed knowledge of the flow. We often are interested in the gross behavior of a device; in such cases it is more appropriate to use integral formulations of the basic laws. An example might be the overall lift a wing produces. Integral formulations, using *finite* systems or control volumes, usually are easier to treat analytically. The basic laws of mechanics and thermodynamics, formulated in terms of finite systems, are the basis for deriving the control volume equations in Chapter 4.

## Methods of Description

We use a method of description that follows the particle when we want to keep track of it. This sometime is referred to as the *Lagrangian* method of description. Consider, for example, the application of

Newton's second law to a particle of fixed mass. Mathematically, we can write Newton's second law for a system of mass  $m$  as

$$\Sigma \vec{F} = m\vec{a} = m \frac{d\vec{V}}{dt} = m \frac{d^2\vec{r}}{dt^2} \quad (1.2)$$

In Eq. 1.2,  $\Sigma \vec{F}$  is the sum of all external forces acting on the system,  $\vec{a}$  is the acceleration of the center of mass of the system,  $\vec{V}$  is the velocity of the center of mass of the system, and  $\vec{r}$  is the position vector of the center of mass of the system relative to a fixed coordinate system. In Example 1.3, we show how Newton's second law is applied to a falling object to determine its speed.

### Example 1.3 FREE FALL OF BALL IN AIR

The air resistance (drag force) on a 200 g ball in free flight is given by  $F_D = 2 \times 10^{-4} V^2$ , where  $F_D$  is in newtons and  $V$  is in meters per second. If the ball is dropped from rest 500 m above the ground, determine the speed at which it hits the ground. What percentage of the terminal speed is the result? (The *terminal speed* is the steady speed a falling body eventually attains.)

**Given:** Ball,  $m = 0.2$  kg, released from rest at  $y_0 = 500$  m.  
Air resistance,  $F_D = kV^2$ , where  $k = 2 \times 10^{-4}$  N·s<sup>2</sup>/m<sup>2</sup>.  
Units:  $F_D$ (N),  $V$  (m/s).

**Find:**

- Speed at which the ball hits the ground.
- Ratio of speed to terminal speed.

**Solution:**

**Governing equation:**  $\Sigma \vec{F} = m\vec{a}$

**Assumption:** Neglect buoyancy force.

The motion of the ball is governed by the equation

$$\Sigma F_y = ma_y = m \frac{dV}{dt}$$

Since  $V = V(y)$ , we write  $\Sigma F_y = m \frac{dV}{dy} \frac{dy}{dt} = mV \frac{dV}{dy}$ . Then,

$$\Sigma F_y = F_D - mg = kV^2 - mg = mV \frac{dV}{dy}$$

Separating variables and integrating,

$$\int_{y_0}^y dy = \int_0^V \frac{mV dV}{kV^2 - mg}$$

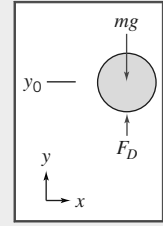
$$y - y_0 = \left[ \frac{m}{2k} \ln(kV^2 - mg) \right]_0^V = \frac{m}{2k} \ln \frac{kV^2 - mg}{-mg}$$

Taking antilogarithms, we obtain

$$kV^2 - mg = -mg e^{[(2k/m)(y-y_0)]}$$

Solving for  $V$  gives

$$V = \left\{ \frac{mg}{k} \left( 1 - e^{[(2k/m)(y-y_0)]} \right) \right\}^{1/2}$$



Substituting numerical values with  $y = 0$  yields

$$V = \left\{ 0.2 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} \times \frac{\text{m}^2}{2 \times 10^{-4} \text{ N} \cdot \text{s}^2} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \left( 1 - e^{[2 \times 2 \times 10^{-4} / 0.2(-500)]} \right) \right\}$$

$$V = 78.7 \text{ m/s} \leftarrow \frac{V}{V_t}$$

At terminal speed,  $a_y = 0$  and  $\Sigma F_y = 0 = kV_t^2 - mg$ .

$$\text{Then, } V_t = \left[ \frac{mg}{k} \right]^{1/2} = \left[ 0.2 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} \times \frac{\text{m}^2}{2 \times 10^{-4} \text{ N} \cdot \text{s}^2} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \right]^{1/2}$$

$$= 99.0 \text{ m/s}$$

The ratio of actual speed to terminal speed is

$$\frac{V}{V_t} = \frac{78.7}{99.0} = 0.795, \text{ or } 79.5\% \leftarrow \frac{V}{V_t}$$

*This problem:*

- Reviewed the methods used in particle mechanics.
- Introduced a variable aerodynamic drag force.

We could use this Lagrangian approach to analyze a fluid flow by assuming the fluid to be composed of a very large number of particles whose motion must be described. However, keeping track of the motion of each fluid particle would become a horrendous bookkeeping problem. Consequently, a particle description becomes unmanageable. Often we find it convenient to use a different type of description. Particularly with control volume analyses, it is convenient to use the field, or *Eulerian*, method of description, which focuses attention on the properties of a flow at a given point in space as a function of time. In the Eulerian method of description, the properties of a flow field are described as functions of space coordinates and time. We shall see in Chapter 2 that this method of description is a logical outgrowth of the assumption that fluids may be treated as continuous media.

## 1.4 Dimensions and Units

Engineering problems are solved to answer specific questions. It goes without saying that the answer must include units, and it is very important to know what the units of a problem are. In 1999, NASA's Mars Climate Observer crashed because the JPL engineers assumed that a measurement was in meters, but the supplying company's engineers had actually made the measurement in feet. Consequently, it is appropriate to present a brief review of dimensions and units.

We refer to physical quantities such as length, time, mass, and temperature as *dimensions*. In terms of a particular system of dimensions, all measurable quantities are subdivided into the two groups of *primary* and *secondary* quantities. We refer to a small group of dimensions from which all others can be formed as primary quantities, and for which we set up arbitrary scales of measure. Secondary quantities are those quantities whose dimensions are expressible in terms of the dimensions of the primary quantities.

*Units* are the arbitrary names (and magnitudes) assigned to the primary dimensions adopted as standards for measurement. For example, the primary dimension of length may be measured in units of meters, feet, yards, or miles. These units of length are related to each other through unit conversion factors such as 1 mile = 5280 feet = 1609 meters.

### Systems of Dimensions

Any valid equation that relates physical quantities must be dimensionally homogeneous; each term in the equation must have the same dimensions. We recognize that Newton's second law relates the four